Information Theoretically Secure Hypothesis Test for Temporally Unstructured Quantum Computing

How Do I Know If You Have A Quantum Computer

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This Presentation

Introduction

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I think I'll buy a quantum computer
Are you sure this seller has one?
Well I can't check the computations with my computer
Is he even sure he has one?
You're always so sceptical. I'll send him a Hypothesis test

A well designed *Hypothesis test* should allow:

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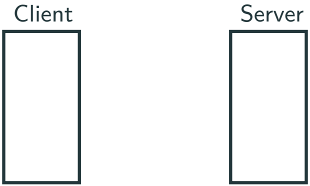
A well designed *Hypothesis test* should allow:

- A Client to ensure a malicious Server is capable of quantum computations.
- An engineer to check their machine is capable of quantum computations.

A Proposal



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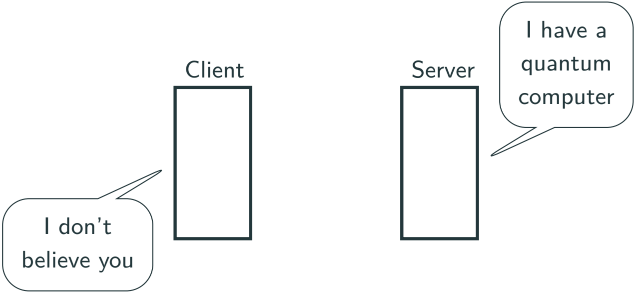

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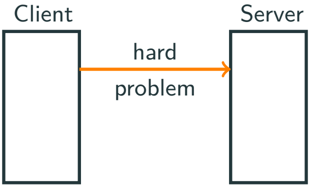

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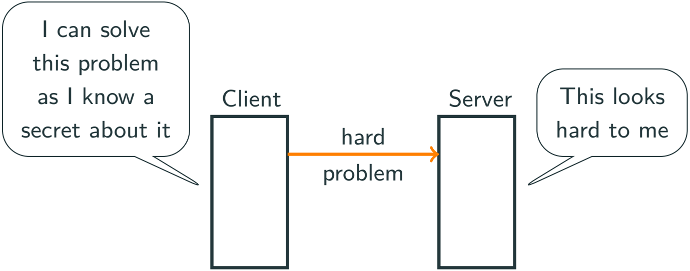
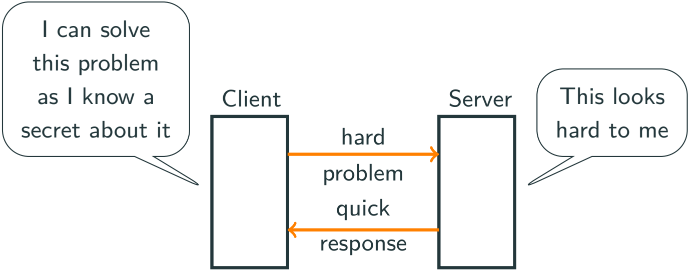

I can solve this problem as I know a secret about it
Client
hard problem
Server
This looks hard to me

A Proposal



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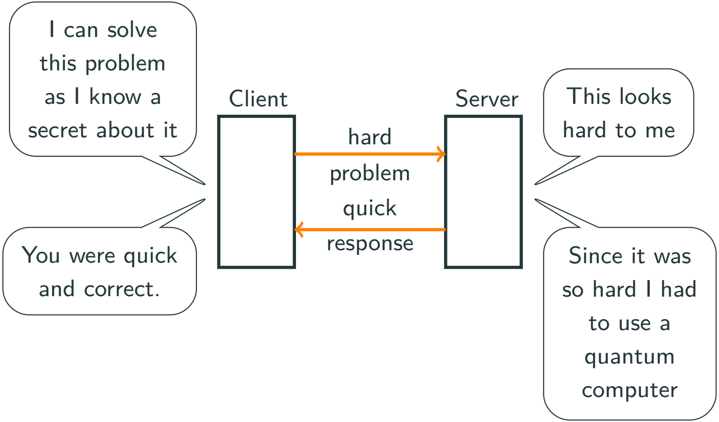
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You were quick and correct.
Since it was so hard I had to use a quantum computer

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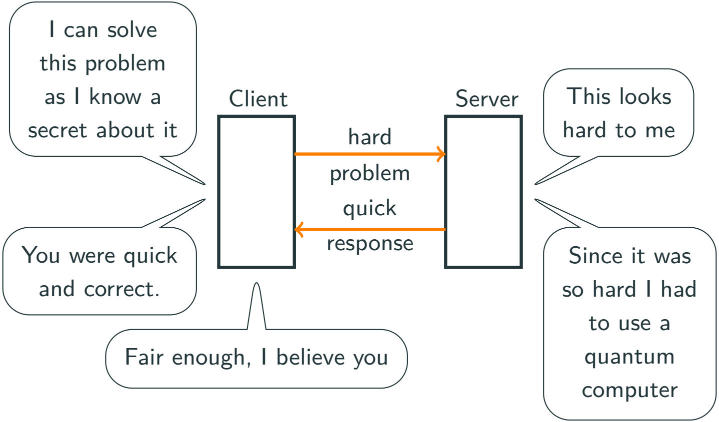
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Client
Server
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hard problem
quick response
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Fair enough, I believe you

A Proposal

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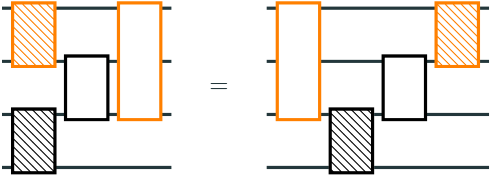
So three conditions for a successful hypothesis test might be:

- The Server must complete a hard computations
  - This means it is capable of computations from some class
- The Client knows a secret property allowing them to check the outcome
  - Must be sure that this does not add structure to the problem which the Server can use
- The Client hides the secret property

IQP in MBQC

Instantaneous Quantum Polytime Machine [Shepherd and Bremner, 2009]

Commuting gates:



In particular:

$$\exp\left\{ i\theta \bigotimes_{i:q_i=1} X_i \right\}$$

where $q \in \{0,1\}^{n_p}$, $\theta \in [0, 2\pi]$.

Instantaneous Quantum Polytime Machine [Shepherd and Bremner, 2009]

$$\exp\left\{ i\theta \bigotimes_{i:q_i=1} X_i \right\}$$

An IQP program may consist of many of these gates, and so many different q. Hence we may represent the whole computation by, for example:

$$\mathbf{Q} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

where, in this case, we have two gates defined by $q = (101)$ and $q = (010)$.

The input is $|0^{n_p}\rangle$ and the output is the resulting state measured in the computational basis.

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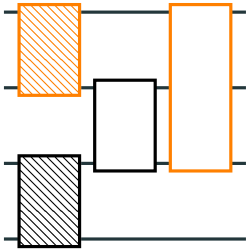
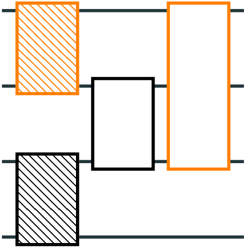
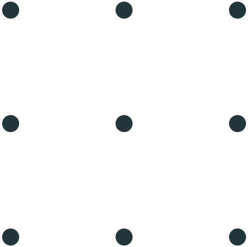
Thought not to be classically simulateable [Bremner et al., 2010]



Measurement Based Quantum Computing [Raussendorf and Briegel, 2001]



vs



$\bigotimes |+\rangle$

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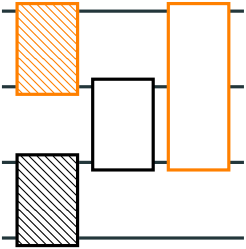

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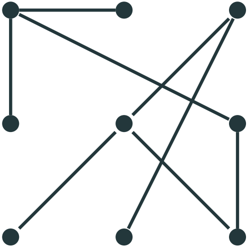

$cZ \ldots cZ \bigotimes |+\rangle$

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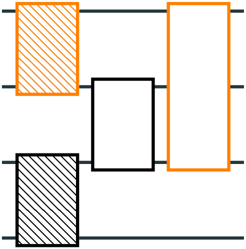

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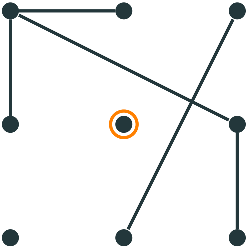

$$|m\rangle \langle m| \, cZ...cZ \bigotimes |+\rangle$$

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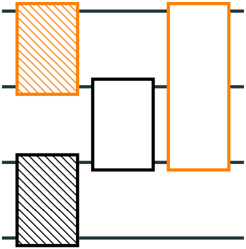

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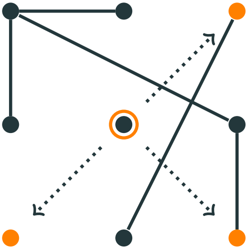

$C \ldots CcZ \ldots cZ \bigotimes |+\rangle$

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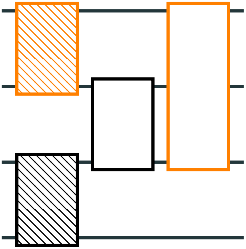

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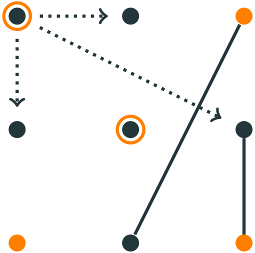

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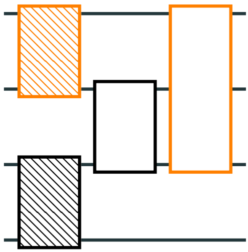

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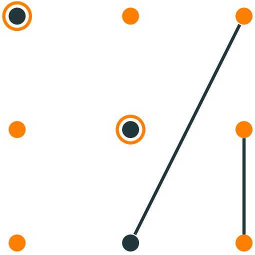

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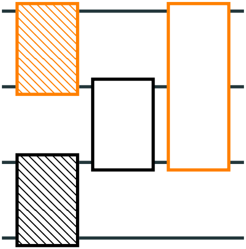

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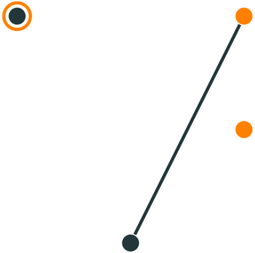
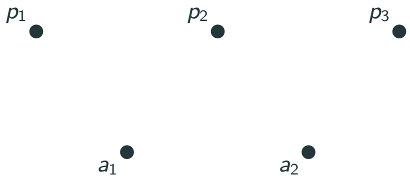


IQP in MBQC



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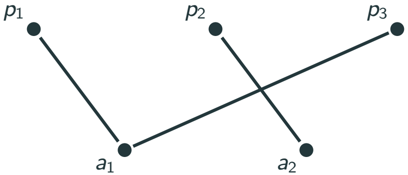

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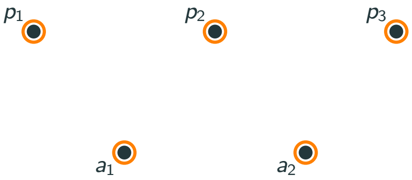

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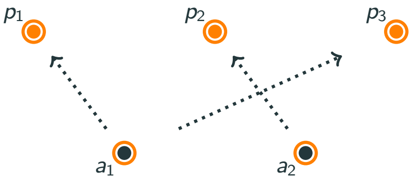

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Blind IQP

Bridge and Break [Fitzsimons and Kashefi, 2012]

$cZ_{1,2}cZ_{2,3} \, |0/1\rangle \otimes |\phi\rangle$

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$$Z_1^{0/1} Z_3^{0/1} |0/1\rangle \otimes |\phi\rangle$$

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$cZ_{1,2}cZ_{2,3}\,|+/-\rangle \otimes |\phi\rangle$

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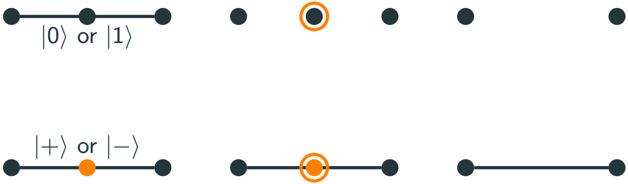
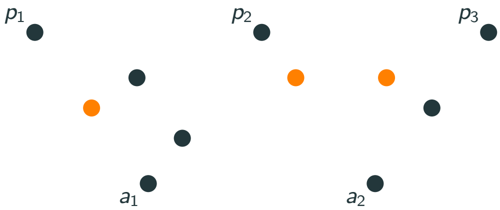


$S_1^{f(+/-,s)} S_3^{f(+/-,s)} Z_{1,3} |\phi\rangle$

IQP By Bridge and Break



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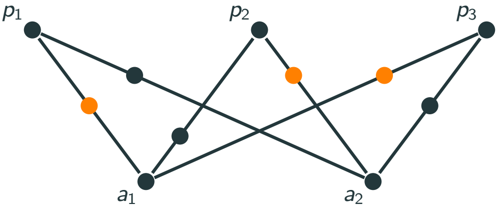

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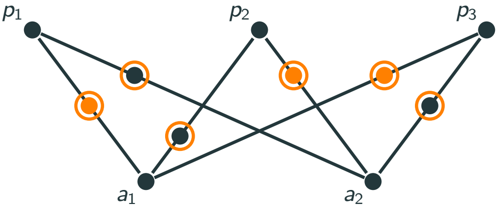

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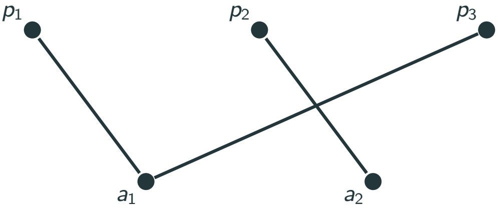

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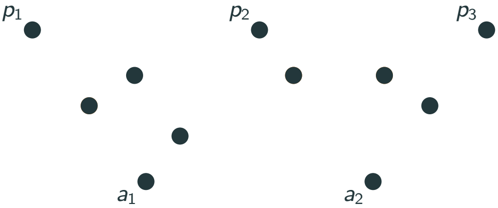

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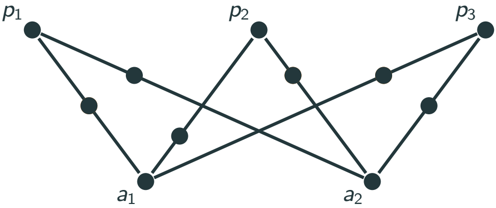

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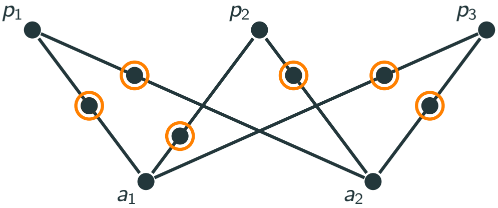

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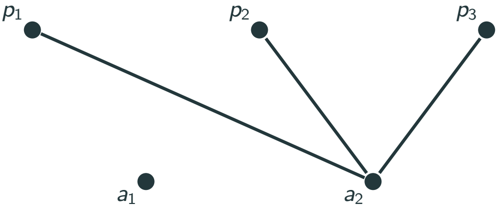

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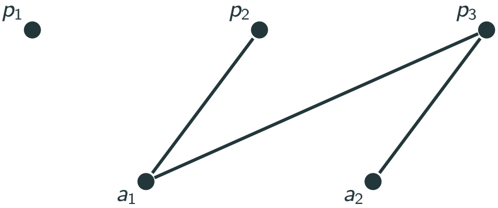

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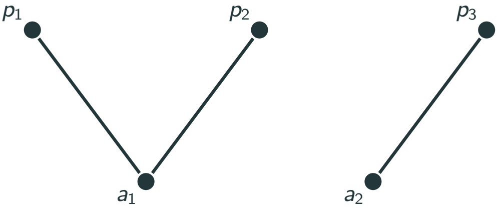
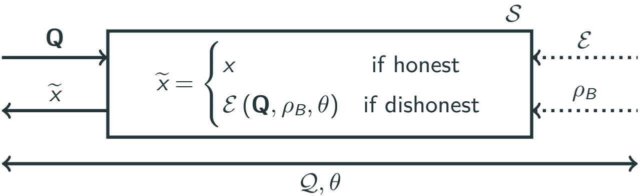
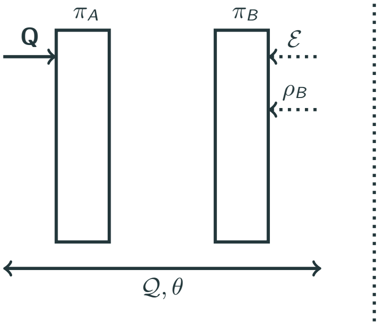


Blind IQP Ideal Resource [Dunjko et al., 2014]



Blind IQP Real Resource



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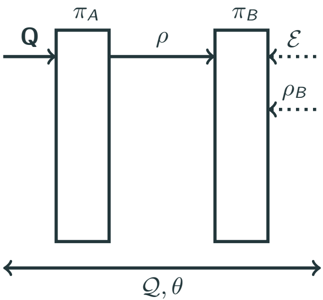

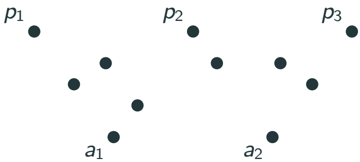

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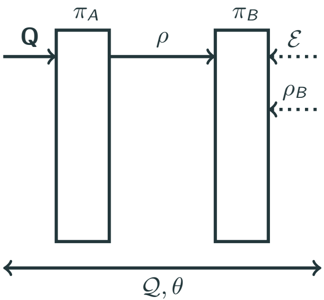

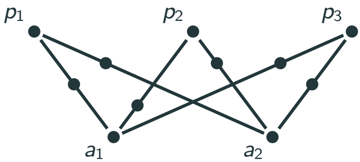

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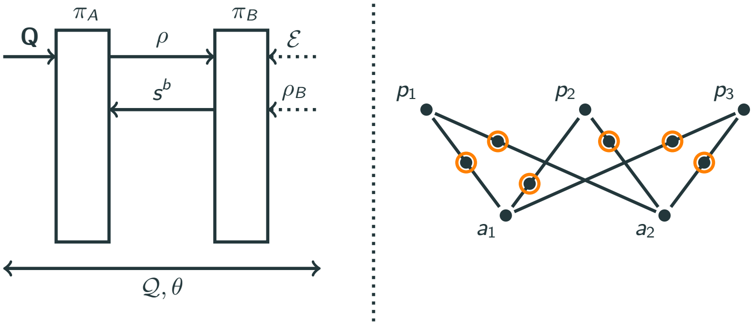

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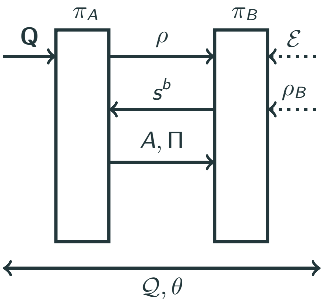

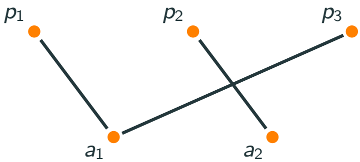

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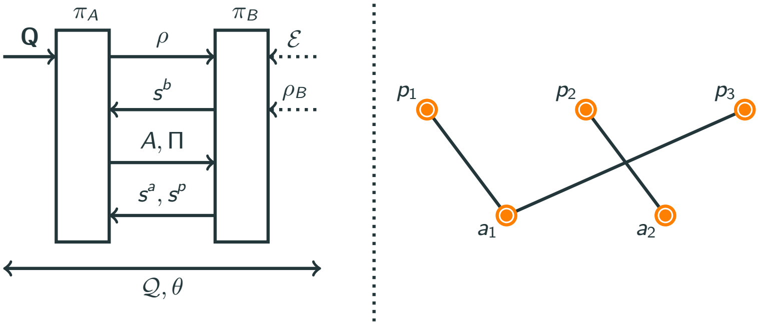

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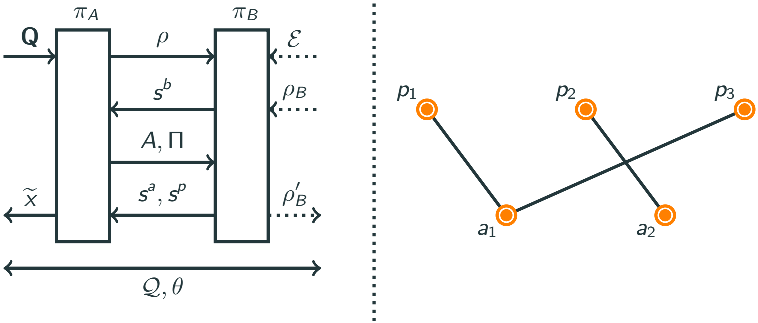
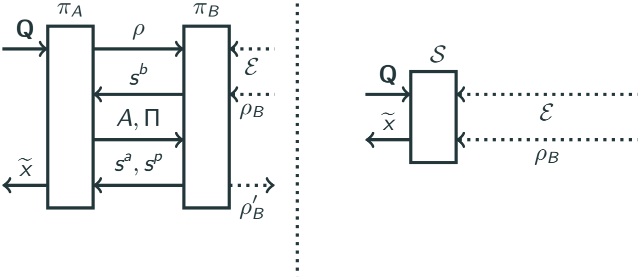


Blind IQP Real vs Ideal Resource



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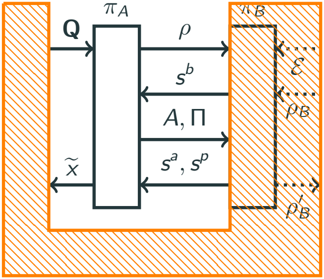

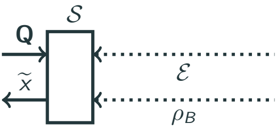

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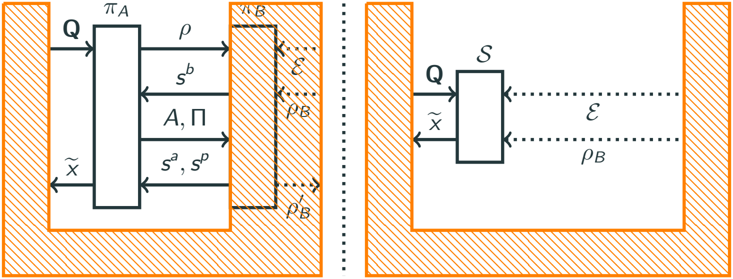

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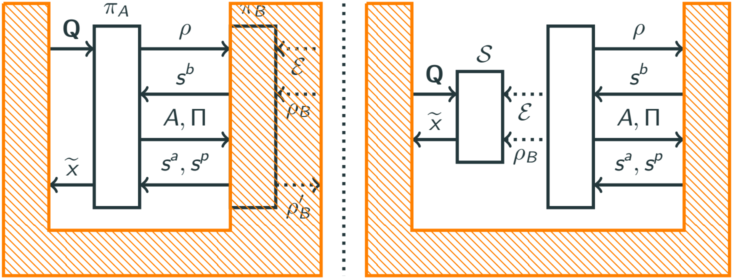


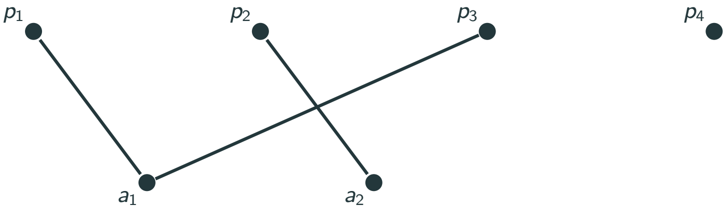
The Hypothesis Test

Bias of a random variable, $X \in \{0,1\}^{n_p}$, in a direction $s \in \{0,1\}^{n_p}$.

$$\mathbb{P}\left(X \cdot s^T = 0\right) = \mathit{Bias}(X, s)$$

Can be easily calculated, for some special IQP computations (depending on s), if one knows s [Shepherd and Bremner, 2009].

Hypothesis Test



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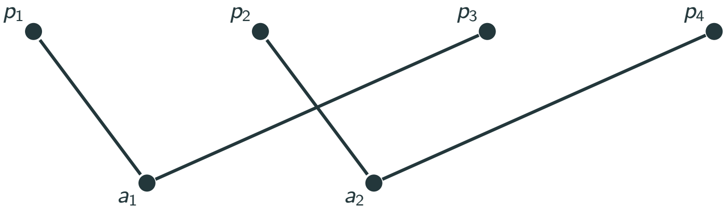
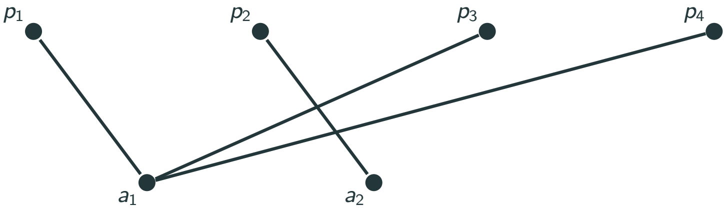


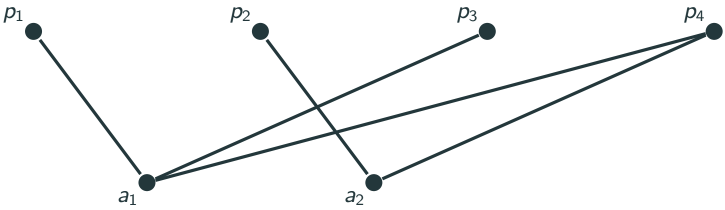
$$Bias(X, s_1) = p$$

Hypothesis Test



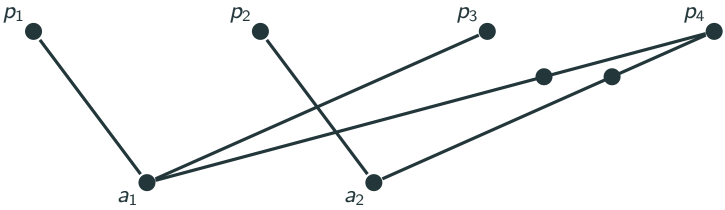
$$Bias(X, s_2) = p$$

Hypothesis Test

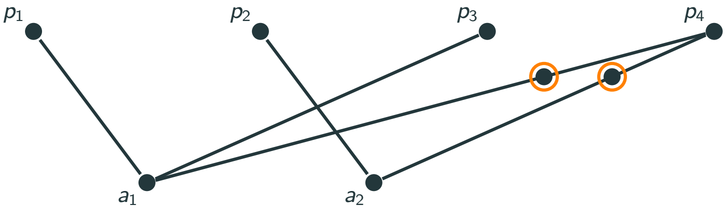


$$Bias(X, s_3) = p$$

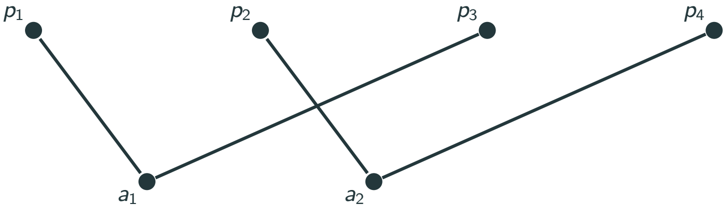
Hypothesis Test



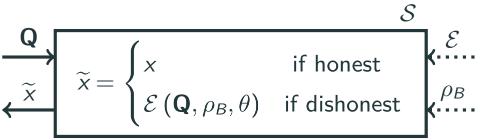
Hypothesis Test



Hypothesis Test



Blind IQP Ideal Resource Used in Hypothesis Test



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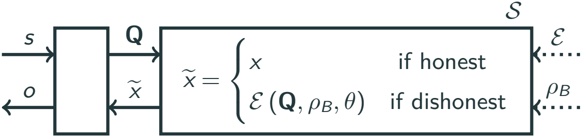

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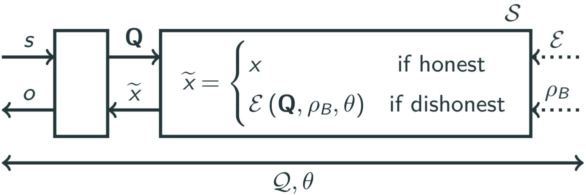
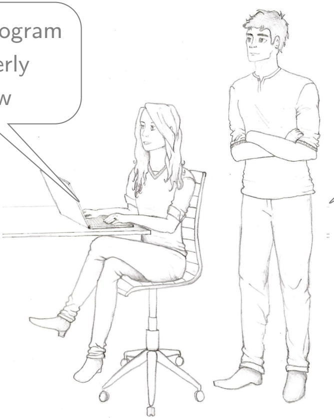


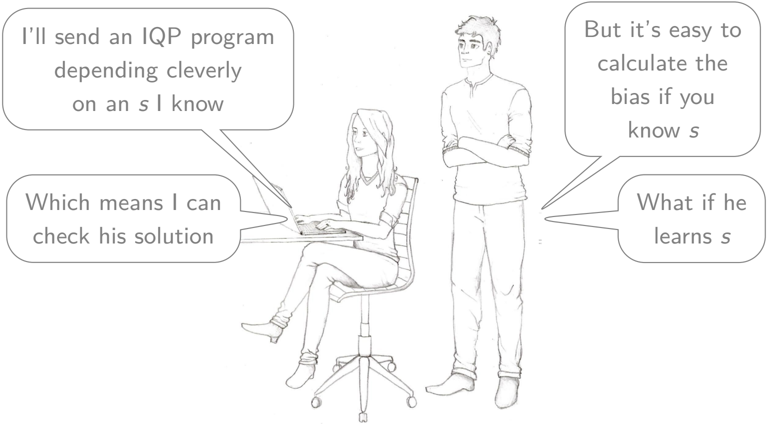
The Hypothesis Test Outline

I'll send an IQP program depending cleverly on an s I know

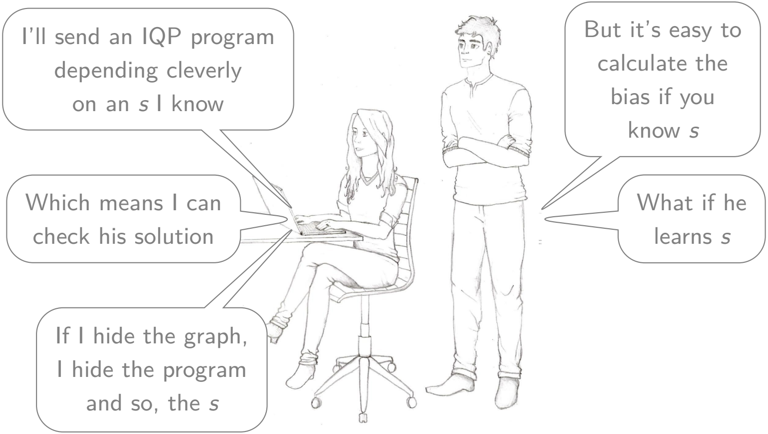


But it's easy to calculate the bias if you know s

The Hypothesis Test Outline



The Hypothesis Test Outline



The Hypothesis Test Outline

I'll send an IQP program depending cleverly on an s I know

Which means I can check his solution

If I hide the graph, I hide the program and so, the s

But it's easy to calculate the bias if you know s

What if he learns s

Seems legit

Three conditions for a successful hypothesis test:

The Hypothesis Test Outline

Three conditions for a successful hypothesis test:

- The Server must complete a hard computations
  - Computation bias calculation is hard

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The Hypothesis Test Outline

Three conditions for a successful hypothesis test:

- The Server must complete a hard computations
  - Computation bias calculation is hard
- The Client knows a secret property allowing them to check the outcome
  - The Client knows the direction s
- The Server hides the secret property
  - Using blind IQP

Closing Remarks

Conclusion:

- Blind IQP computation
- Can hide computable computation in hard one
- Might allow demonstration of quantum supremacy in early devices

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The paper:

arxiv.org/abs/1704.01998

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